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LETTER TO THE EDITOR

Induced Chern–Simons action in a dissipative medium

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Abstract. We study the influence of finite particle lifetime τ (dissipation) on the induced Chern–Simons term in a heated and dense medium. We demonstrate that dissipation like a temperature suppresses topological action. A peculiarity in introducing a dissipation into the theory is discussed briefly.

Recently, (2+1)-dimensional Chern–Simons (CS) theories [1] have been a subject of intensive studies. An important feature of such models is that, although invariant under continuous gauge transformations, they break both parity and time reversal symmetries. Most notably, a CS term appears in a variety of effective-field theories resulting from integrating out massive two-component fermions, since the mass term, $m\bar{\Psi}\Psi$, in two spatial dimensions is odd under P and T transformations [2].

If we start with the QED$_3$ Lagrangian

$$L=\bar{\Psi}(\mathrm{i}\partial_\mu\gamma^\mu-eA_\mu\gamma^\mu-m)\Psi \tag{1}$$

then the CS term

$$L_{\mathrm{CS}}=\pm\frac{e^2}{8\pi}I(m)\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \tag{2}$$

is generated quantum mechanically at the 1-loop level. Here $\pm$ corresponds to the sign of the mass m (for the sake of clarity, we consider $m>0$) and the factor

$$I(m)=\frac{m}{\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{\omega^2+m^2}=1 \tag{3}$$

ω being the zeroth component of the Euclidean momentum. This result is proven to persist to all orders in perturbation theory [3].

Another motivation for studying the Abelian CS theory (equations (2), (3)) is its direct application to solid state physics, including the quantum Hall effect [4], planar quantum antiferromagnets [5], and anyonic systems (for a review of anyonic superconductivity, see e.g. [6]).

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In condensed matter, thermodynamic parameters such as temperature, T, chemical potential, μ, etc, should be taken into account. They are known to renormalize the CS action [7, 8]

$$I(m;\mu,T)=\frac{1}{2}\left(\tanh\frac{m+\mu}{2T}+\tanh\frac{m-\mu}{2T}\right) \tag{4}$$

which is important for the anyonic systems [6, 8].

It also seems instructive to study the system (2) away from equilibrium, introducing dissipation. As a starting point we consider in this letter the simplest form of dissipation (friction) assuming that the fermions have a finite lifetime τ. This assumption, while relevant to non-equilibrium states, is at least consistent with the Matsubara finite-temperature technique. Note also that it is precisely in this way that magnetic impurities affect a superconducting phase [9]. We show below that the dissipation acts much like a finite temperature both destroying coherent effects and suppressing quantum fluctuations.

The finite lifetime $\tau=1/\eta$ is introduced by shifting the quasiparticle energy to the complex plane: $p_0\to p_0+\mu+\mathrm{i}\,\mathrm{sgn}(p_0+\mu)$. If properly Wick rotated ($p_0\to \mathrm{i}\omega$), the substitution into (3) becomes: $\omega\to\omega-\mathrm{i}\mu+\eta\,\mathrm{sgn}(\omega)$. Note that in the Euclidean space it looks like implying the finite lifetime for underlying fermions, not the quasiparticles. In reality, it is the dynamics of the propagating quasiparticles that should be dissipative. The assumption that underlying fermions have a finite lifetime makes the Matsubara technique ambiguous. As a consequence for bosonic systems, one should not imply the dissipation before extracting a Bose condensate.

Making use of the digamma function $\psi(z)$, we get a rather compact final expression

$$I(m;\mu,T,\eta)=\frac{1}{\pi}\Im\psi\left(\frac{\eta}{2\pi T}+\frac{1}{2}+\mathrm{i}\frac{\mu+m}{2\pi T}\right)-\frac{1}{\pi}\Im\psi\left(\frac{\eta}{2\pi T}+\frac{1}{2}+\mathrm{i}\frac{\mu-m}{2\pi T}\right) \tag{5}$$

for the CS action in a dissipative medium. Clearly, equation (5) coincides with (4) when $\eta=0$.

In order to gain further insight we simplify equation (5) setting $T=0$:

$$I(m;\mu,\eta)=\frac{1}{\pi}\left(\tan^{-1}\frac{m+\mu}{\eta}+\tan^{-1}\frac{m-\mu}{\eta}\right). \tag{6}$$

Loss of quantum coherence can readily be seen in the smoothening of the step-function profile of

$$I(m;\mu,\eta=0)=\Theta(m^2-\mu^2) \tag{7}$$

much like in the case of finite temperature. However, dissipative corrections enter usually as a power law while the (low-) temperature effects have an exponential falloff. Moreover, at finite η, the low-temperature corrections are amplified and acquire a power-law behaviour as well (see also [10]).

To observe the dissipative suppression of quantum fluctuations, it suffices to consider vacuum effects ($T=\mu=0$). In this case, the factor $I(m;\mu)$ in the quantum-mechanically induced CS term (2) reduces from 1 to

$$I(m;\eta)=\frac{2}{\pi}\tan^{-1}\frac{m}{\eta} \tag{8}$$

which is the pure manifestation of the destructive action of the dissipation.

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